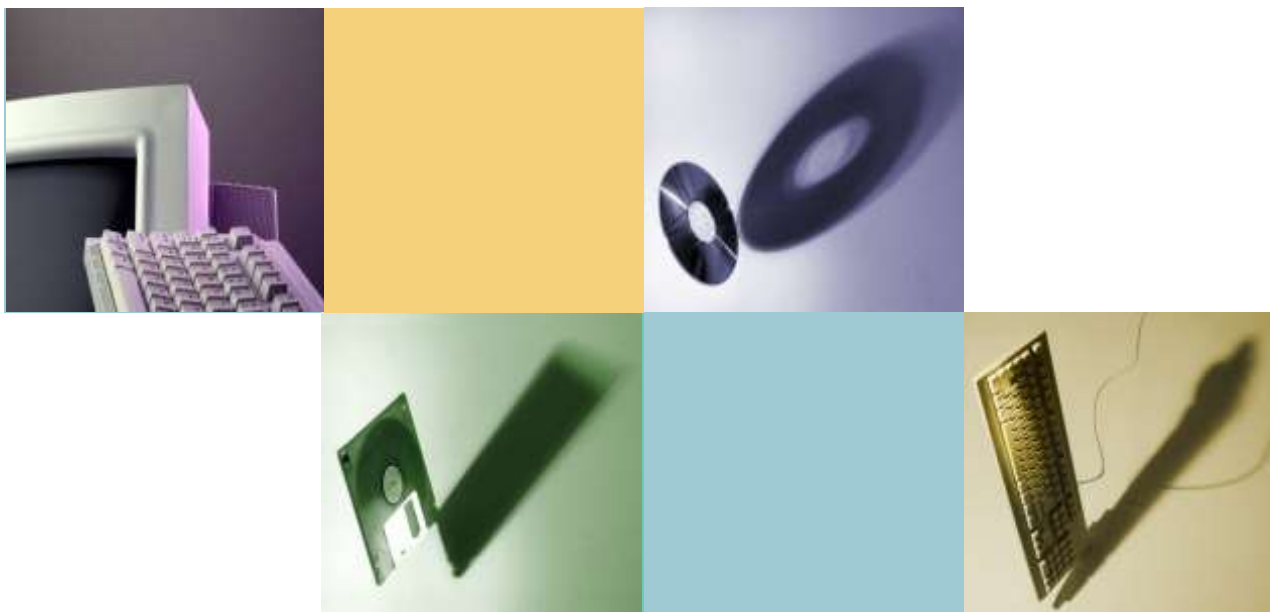


Diferenciación e Integración de Funciones Exponenciales y Logarítmicas



Cálculo



Actividades 3.4

- **Referencia del Texto:** Sección 5.1 La Función Logarítmica: Ver ejemplos 5, 6, 8; Ejercicios de Práctica: Impares 41-63, 65-69; Sección 5.2 – La Función Logaritmo natural: Ver impares 1 al 10; Ejercicios de Práctica: Impares 1- 25; Sección 5.4 Funciones Exponenciales: Derivación e Integración: Ver Ejemplos 3 y 4
- **Referencias del Web:**
- Math2me: [Derivar una Función Exponencial](#); [Derivar una Función Exponencial con base constant](#); [Derivación Logarítmica](#); [Derivación de una Función Logaritmo Natural](#); [Derivación de una Función Logaritmo Natural – Ejercicio 1](#); [Integrales definidas trascendentales](#)
- Julio Profe – [Reglas para deriver funciones Logarítmicas](#); Derivación Logarítmica; Derivación Logarítmica – [Ejercicio 1](#); Integrales por sustitución – [Ejercicio 7](#); [Ejercicio 8](#)



Derivadas de la funciones exponenciales y logarítmicas

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Si u es una función de x ...

$$\frac{d}{dx}(a^u) = a^u \ln a \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$$



Ejemplo 1 & 2

- Calcule $\frac{dy}{dx}$ si

$$y = \ln(2x^3 + \pi^2)$$

$$\frac{dy}{dx} = \frac{d}{dx} \ln(2x^3 + \pi^2)$$

$$\frac{dy}{dx} = \frac{1}{2x^3 + \pi^2} \cdot \frac{d}{dx} (2x^3 + \pi^2)$$

$$\frac{dy}{dx} = \frac{6x^2}{2x^3 + \pi^2}$$

$$y = \log(1 + \cos x)$$

$$\frac{dy}{dx} = \frac{d}{dx} \log(1 + \cos x)$$

$$= \frac{1}{(1 + \cos x) \ln 10} \frac{d}{dx} (1 + \cos x)$$

$$= \frac{-\sin x}{(1 + \cos x) \ln 10}$$



Ejemplo 3

Calcule

$$\frac{d}{dt} \left(e^{x^{12} \ln x} \right)$$

Regla de la Cadena

$$= e^{x^{12} \ln x} \frac{d}{dx} (x^{12} \ln x)$$

Regla del Producto

$$= e^{x^{12} \ln x} \left(x^{12} \cdot \frac{d}{dx} \ln x + \ln x \cdot \frac{d}{dx} x^{12} \right)$$

$$= e^{x^{12} \ln x} \left(x^{12} \cdot \frac{1}{x} + \ln x \cdot 12x^{11} \right)$$

$$= e^{x^{12} \ln x} (x^{11} + \ln x \cdot 12x^{11})$$

$$= x^{11} (1 + 12 \ln x) e^{x^{12} \ln x}$$



Diferenciación Logarítmica

- Calcule $\frac{dy}{dx}$ si :

$$y = x^5$$

$$y = 5^x$$

$$y = x^x$$

$$\frac{dy}{dx} = 5x^4$$

$$\frac{dy}{dx} = 5^x \ln 5$$

$$\frac{dy}{dx} = ?$$



Ejemplo 4

- Calcule $\frac{dy}{dx}$

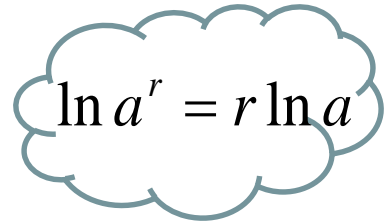
$$y = x^x$$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} x \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = x \frac{d}{dx} \ln x + \ln x$$


$$\ln a^r = r \ln a$$

$$\frac{1}{y} \frac{dy}{dx} = 1 + \ln x$$

$$\frac{dy}{dx} = y(1 + \ln x)$$

$$= x^x (1 + \ln x)$$



Ejemplo 5

- Calcule $\frac{dy}{dx}$ si $y = x^{\sqrt{x}} e^{x-1}$
- Solución:

$$\ln y = \ln x^{\sqrt{x}} e^{x-1}$$

$$\ln y = \ln x^{\sqrt{x}} + \ln e^{x-1}$$

$$\ln y = \sqrt{x} \ln x + (x-1) \ln e$$

$$\ln y = \sqrt{x} \ln x + x - 1$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} (\sqrt{x} \ln x + x - 1)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx} (\sqrt{x} \ln x) + 1$$

$$\frac{1}{y} \frac{dy}{dx} = \sqrt{x} \cdot \frac{1}{x} + \ln x \cdot \frac{1}{2\sqrt{x}} + 1$$

$$\frac{dy}{dx} = y \left[\frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}} + 1 \right]$$

$$= x^{\sqrt{x}} e^{x-1} \left[\frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}} + 1 \right]$$



Ejercicio #3

- Calcule $\frac{dy}{dx}$ si $y = (1 + 5x)^{\frac{1}{x}}$

$$\ln y = \ln (1 + 5x)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \ln (1 + 5x)$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} \left[\frac{1}{x} \ln (1 + 5x) \right]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} \frac{d}{dx} \ln (1 + 5x) + \ln (1 + 5x) \frac{d}{dx} \frac{1}{x}$$



Ejercicio #3 ...

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} \cdot \frac{5}{1+5x} + \ln(1+5x) \cdot \frac{-1}{x^2}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{5}{x(1+5x)} - \frac{\ln(1+5x)}{x^2}$$

$$\frac{dy}{dx} = y \left[\frac{5}{x(1+5x)} - \frac{\ln(1+5x)}{x^2} \right]$$

$$= (1+5x)^{\frac{1}{x}} \left[\frac{5}{x(1+5x)} - \frac{\ln(1+5x)}{x^2} \right]$$



Fórmulas de Integración

- Recuerde:

$$\int \frac{1}{x} dx = \ln |x| + c$$

$$\int e^x dx = e^x + c$$

Si a es un número, $\int a^x dx = \frac{a^x}{\ln a} + c$



Ejemplo 6

$$\int 2^x \sqrt{3} \, dx = \sqrt{3} \int 2^x \, dx = \frac{2^x \sqrt{3}}{\ln 2} + c$$

$$\int -3 \cdot \pi^x \, dx = -3 \int \pi^x \, dx = -3 \cdot \frac{\pi^x}{\ln \pi} + c = \frac{-3\pi^x}{\ln \pi} + c$$

$$\int \left(1 - \frac{6}{x} \right) dx = \int dx - 6 \int \frac{1}{x} dx = x - 6 \ln |x| + c$$



Resumen de Fórmulas de Integración

$$\int e^x dx = e^x + c$$

$$\int \csc^2 x dx = -\cot x + c$$

$$\int \frac{1}{x} dx = \ln |x| + c$$

$$\int \sec^2 x dx = \tan x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int \sec x \tan x dx = \sec x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \csc x \cot x dx = -\csc x + c$$

$$\int \cos x dx = \sin x + c$$



Ejercicios del Texto

Finding a Derivative In Exercises 41–64, find the derivative of the function.

41. $f(x) = \ln(3x)$

43. $g(x) = \ln x^2$

45. $y = (\ln x)^4$

47. $y = \ln(t + 1)^2$

49. $y = \ln(x\sqrt{x^2 - 1})$

51. $f(x) = \ln\left(\frac{x}{x^2 + 1}\right)$

53. $g(t) = \frac{\ln t}{t^2}$

55. $y = \ln(\ln x^2)$

57. $y = \ln\sqrt{\frac{x+1}{x-1}}$

59. $f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

61. $y = \ln|\sin x|$

42. $f(x) = \ln(x - 1)$

44. $h(x) = \ln(2x^2 + 1)$

46. $y = x^2 \ln x$

48. $y = \ln\sqrt{x^2 - 4}$

50. $y = \ln[t(t^2 + 3)^3]$

52. $f(x) = \ln\left(\frac{2x}{x+3}\right)$

54. $h(t) = \frac{\ln t}{t}$

56. $y = \ln(\ln x)$

58. $y = \ln\sqrt[3]{\frac{x-1}{x+1}}$

60. $f(x) = \ln(x + \sqrt{4+x^2})$

62. $y = \ln|\csc x|$

Finding an Equation of a Tangent Line In Exercises 65–72, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

65. $y = \ln x^4, (1, 0)$

66. $y = \ln x^{3/2}, (1, 0)$

67. $f(x) = 3x^2 - \ln x, (1, 3)$

68. $f(x) = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right), (0, 4)$

69. $f(x) = \ln\sqrt{1 + \sin^2 x}, \left(\frac{\pi}{4}, \ln\sqrt{\frac{3}{2}}\right)$

70. $f(x) = \sin 2x \ln x^2, (1, 0)$

71. $f(x) = x^3 \ln x, (1, 0)$

72. $f(x) = \frac{1}{2}x \ln x^2, (-1, 0)$



Ejercicios del Texto ...

Finding an Indefinite Integral In Exercises 1–26, find the indefinite integral.

1. $\int \frac{5}{x} dx$

3. $\int \frac{1}{x+1} dx$

5. $\int \frac{1}{2x+5} dx$

7. $\int \frac{x}{x^2-3} dx$

9. $\int \frac{4x^3+3}{x^4+3x} dx$

11. $\int \frac{x^2-4}{x} dx$

13. $\int \frac{x^2+2x+3}{x^3+3x^2+9x} dx$

15. $\int \frac{x^2-3x+2}{x+1} dx$

17. $\int \frac{x^3-3x^2+5}{x-3} dx$

2. $\int \frac{10}{x} dx$

4. $\int \frac{1}{x-5} dx$

6. $\int \frac{9}{5-4x} dx$

8. $\int \frac{x^2}{5-x^3} dx$

10. $\int \frac{x^2-2x}{x^3-3x^2} dx$

12. $\int \frac{x^3-8x}{x^2} dx$

14. $\int \frac{x^2+4x}{x^3+6x^2+5} dx$

16. $\int \frac{2x^2+7x-3}{x-2} dx$

18. $\int \frac{x^3-6x-20}{x+5} dx$

Finding a Derivative In Exercises 33–54, find the derivative.

33. $f(x) = e^{2x}$

35. $y = e^{\sqrt{x}}$

37. $y = e^{x-4}$

39. $y = e^x \ln x$

41. $y = x^3 e^x$

43. $g(t) = (e^{-t} + e^t)^3$

45. $y = \ln(1 + e^{2x})$

34. $y = e^{-8x}$

36. $y = e^{-2x^3}$

38. $y = 5e^{x^2+5}$

40. $y = xe^{4x}$

42. $y = x^2 e^{-x}$

44. $g(t) = e^{-3/t^2}$

46. $y = \ln\left(\frac{1+e^x}{1-e^x}\right)$

Finding an Indefinite Integral In Exercises 91–108, find the indefinite integral.

91. $\int e^{5x}(5) dx$

93. $\int e^{2x-1} dx$

95. $\int x^2 e^{x^3} dx$

92. $\int e^{-x^4}(-4x^3) dx$

94. $\int e^{1-3x} dx$

96. $\int e^x(e^x+1)^2 dx$

