Solución a problemas 26-30 del Examen Parcial 3

26. Encuentre la derivada de y con respecto a x: $y = e^{(10\sqrt{x}+x^3)}$

Solución:

$$\ln y = \ln e^{(10\sqrt{x}+x^3)}$$

$$\ln y = \left(10\sqrt{x} + x^3\right)\ln e$$

$$\ln y = 10\sqrt{x} + x^3$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}(10\sqrt{x} + x^3)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 10\frac{d}{dx}\sqrt{x} + 3x^2$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 10\cdot\frac{1}{2\sqrt{x}} + 3x^2$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{5\sqrt{x}} + 3x^2$$

$$\frac{dy}{dx} = y\cdot\left(\frac{1}{5\sqrt{x}} + 3x^2\right)$$

$$\frac{dy}{dx} = e^{(10\sqrt{x}+x^3)}\cdot\left(\frac{1}{5\sqrt{x}} + 3x^2\right)$$

27) Evalúe el integral

$$\int \frac{x\,dx}{(7x^2+3)^5}$$

$$= \int (7x^2+3)^{-5}x\,dx$$

Sea $u = 7x^2 + 3$

$du = 14x\,dx$

$$= \frac{1}{14}\int (7x^2+3)^{-5}\cdot 14x\,dx$$

$$= \frac{1}{14}\int u^{-5}du$$

$$= \frac{1}{14}\cdot\frac{u^{-4}}{-4} + c$$

$$= -\frac{1}{56}\cdot u^{-4} + c$$

$$= -\frac{1}{56} \cdot (7x^2 + 3)^{-4} + c$$

28) Evalúe el integral

$$\int \frac{dx}{x \ln x^5}$$

$$= \int \frac{1}{\ln x^5} \cdot \frac{1}{x}\, dx$$

$$= \int \frac{1}{5 \ln x} \cdot \frac{1}{x}\, dx$$

$$= \frac{1}{5} \int \frac{1}{\ln x} \cdot \frac{1}{x}\, dx$$

Sea $u = \ln x$

$du = \frac{1}{x} dx$

$$= \frac{1}{5} \int \frac{1}{u} \cdot \frac{1}{x}\, dx$$

$$= \frac{1}{5} \int u^{-1} \cdot du$$

$$= \frac{1}{5} \cdot \ln |u| + c$$

$$= \frac{1}{5} \cdot \ln |\ln x| + c$$

29) Evalúe el integral

$$\int 3x^2 \sqrt[4]{8 + 3x^3}\, dx$$

$$= \int (8 + 3x^3)^{\frac{1}{4}}\, 3x^2\, dx$$

Sea $u = 8 + 3x^3$

$du = 9x^2 dx$

$$= \frac{1}{3} \int (8 + 3x^3)^{\frac{1}{4}} \cdot 3 \cdot 3x^2\, dx$$

$$= \frac{1}{3} \int u^{\frac{1}{4}} \cdot du$$

$$= \frac{1}{3} \cdot \frac{u^{\frac{5}{4}}}{\frac{5}{4}} + c$$

$$= \frac{4}{15} \cdot u^{\frac{5}{4}} + c$$

$$= \frac{4}{15} \cdot (8 + 3x^3)^{\frac{5}{4}} + c$$

30) Evalúe el integral

$$\int \frac{\log_2 x}{x} dx$$

$$= \int \log_2 x \cdot \frac{1}{x} \, dx$$

Sea $u = \log_2 x$

$$du = \frac{1}{x \, ln2} dx$$

$$= \ln 2 \cdot \int \log_2 x \cdot \frac{1}{x \, ln \, 2} \, dx$$

$$= \ln 2 \cdot \int u \cdot du$$

$$= \ln 2 \cdot \frac{u^2}{2} + c$$

$$= \ln 2 \cdot \frac{(\log_2 x)^2}{2} + c$$

$$= \frac{\ln 2 \, (\log_2 x)^2}{2} + c$$