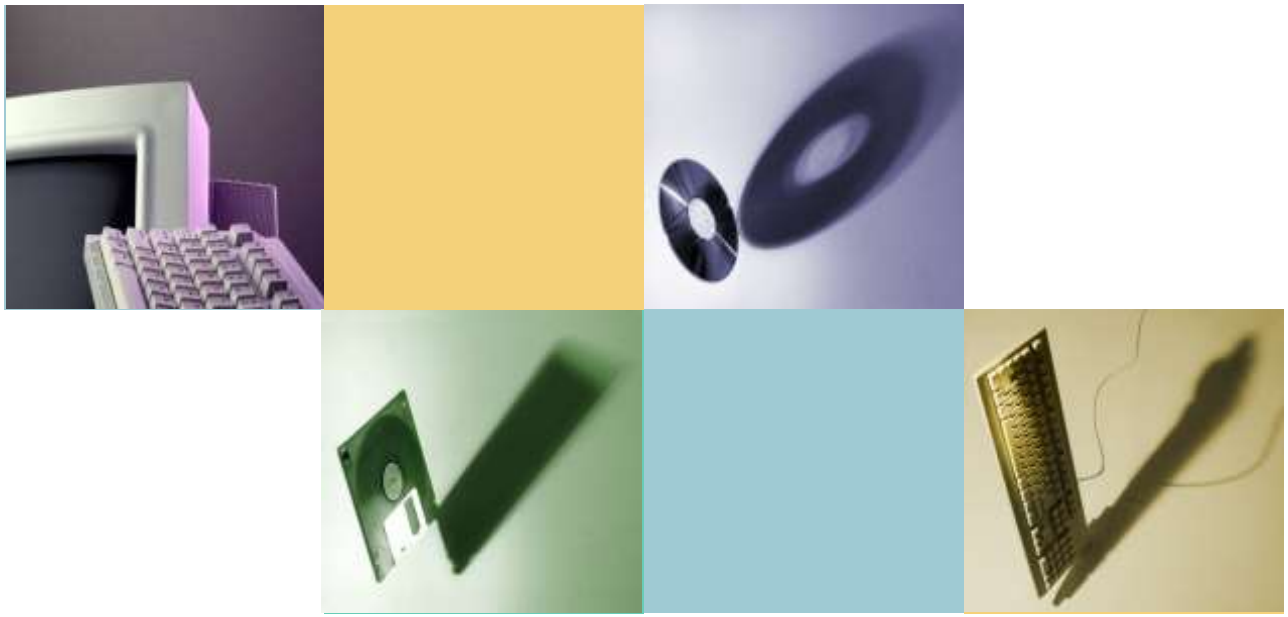
Unidad 3 – Lección 3.2



La Función y su gráfica



Actividades 3.2

- **Referencias en el Texto:**
  - Sección 2.2 – Gráfica de Funciones: 2-9, 33-35, 39-41, 49, 51, 52, 59, 53-56, 57-67 (impares);
  - Sección: 2.3 - 31-36
  - Sección 2.6 – 1-4; 7-13; 19-37, 41-51, 53-66, 69; 83-88
- **Referencias del Web**
  - Gráfica de la Función Lineal – Ejemplo 1; Gráfica de la Función Cuadráticas; Gráfica de una función por trozos; ¿Cuál gráfica representa una función?
  - Graficando con Desmos: Funciones por partes
  - Math2Me – Como hacer traslado de Funciones Valor absoluto); Traslación de una parábola Ejercicio 1; Traslación de una parábola;

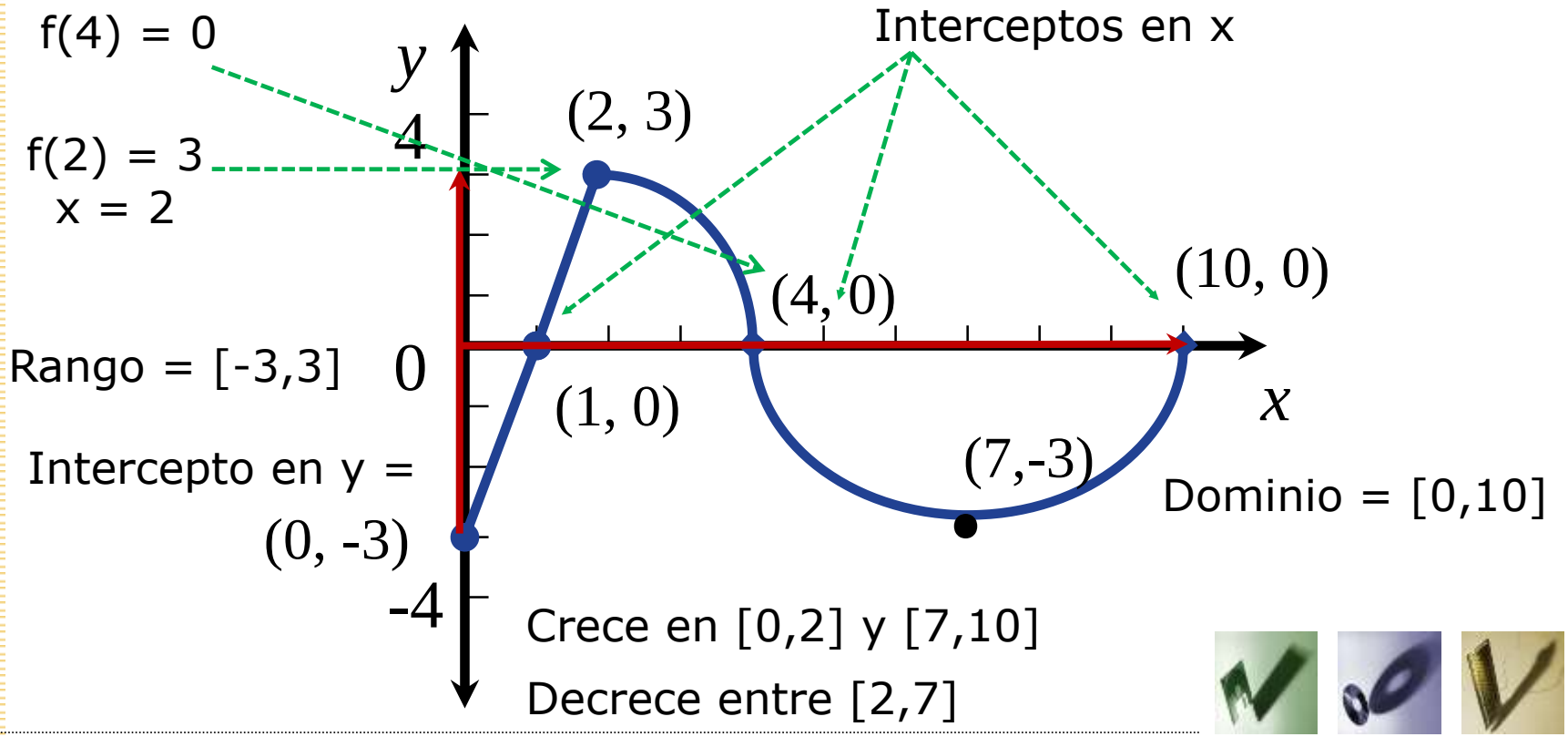


Interpretación de la gráfica

De la gráfica de la función f siguiente, determine

a) $f(4)$.
b) El valor de x, si que $f(x) = 3$
c) El dominio, recorrido e interceptos.
d) Dónde crece y decrece
e) El valor máximo y mínimo de la función.

El valor máximo de la función es 3.
El valor mínimo de la función es -3.



Ejemplo 1

- Para cada una encuentre:

a) Dominio

b) Rango

c) Interceptos

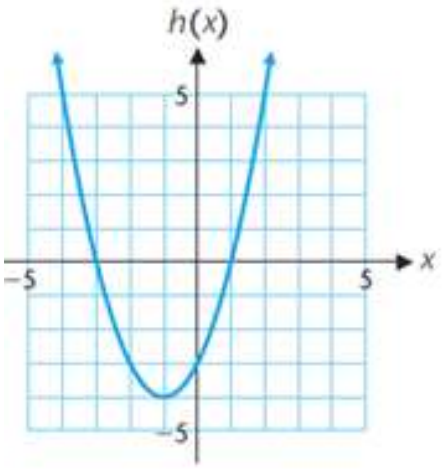
d) donde crece y decrece

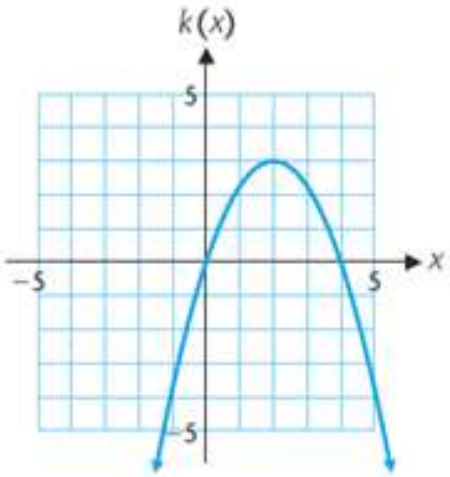
e) Intervalos en dónde es constante

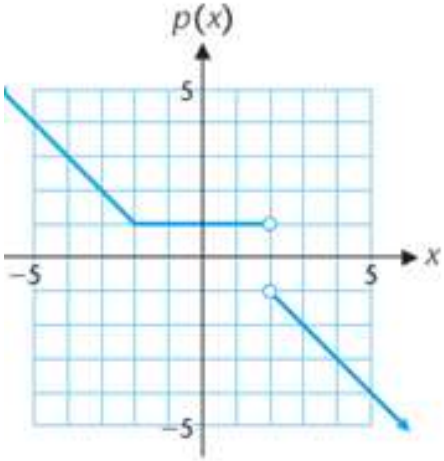
f) Punto de “discontinuidad”

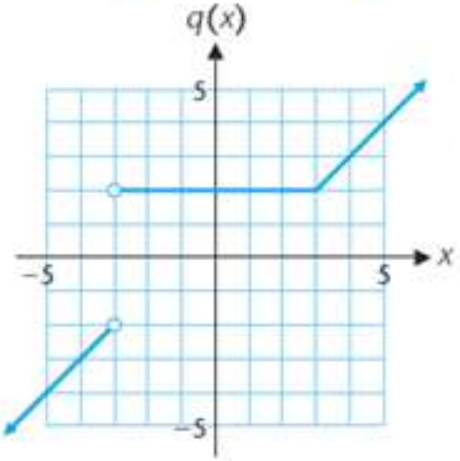
g) Encuentre (si existe):

- q(-5), q(0), q(3)
- p(-3), p(0), p(2)









Graficando funciones

- Tabla de valores
  - Seleccione entre dos a cinco valores para x y determine los valores $f(x)$ correspondientes

Funciones lineales:

$f(x) = mx + b \Leftrightarrow y = mx + b$

Ejemplo:

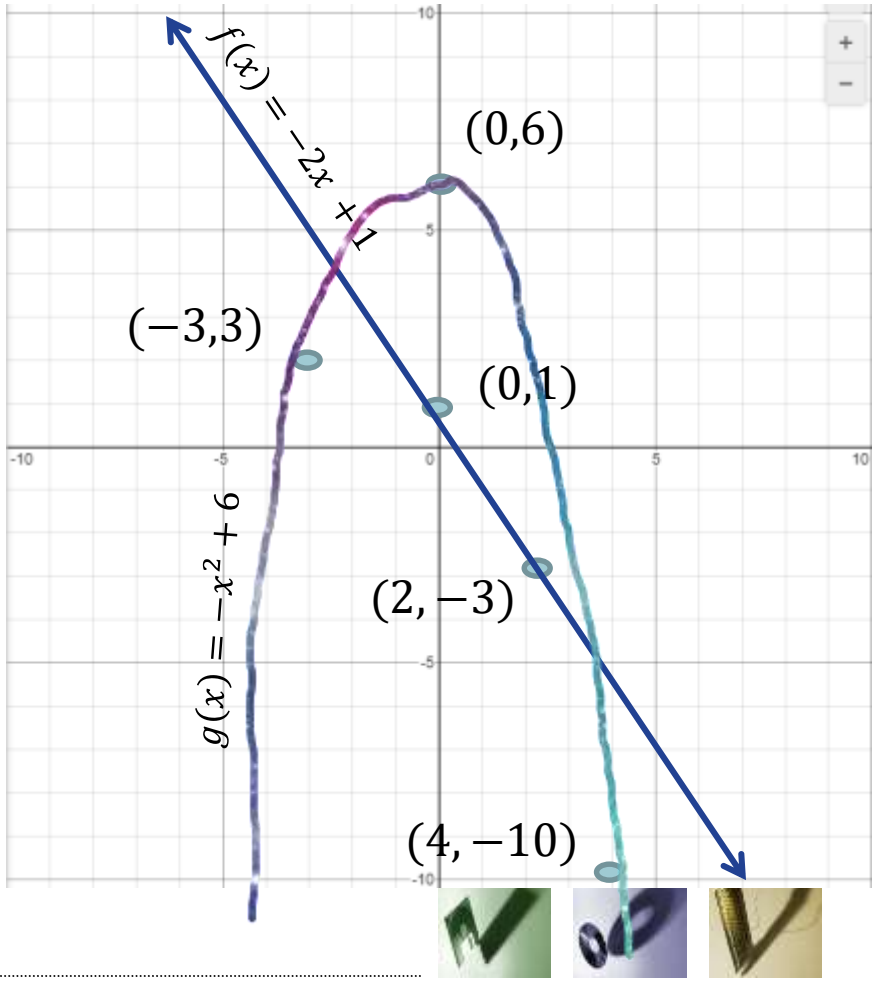
$f(x) = -2x + 1$

x	y
2	-3
0	1

Ejemplo:

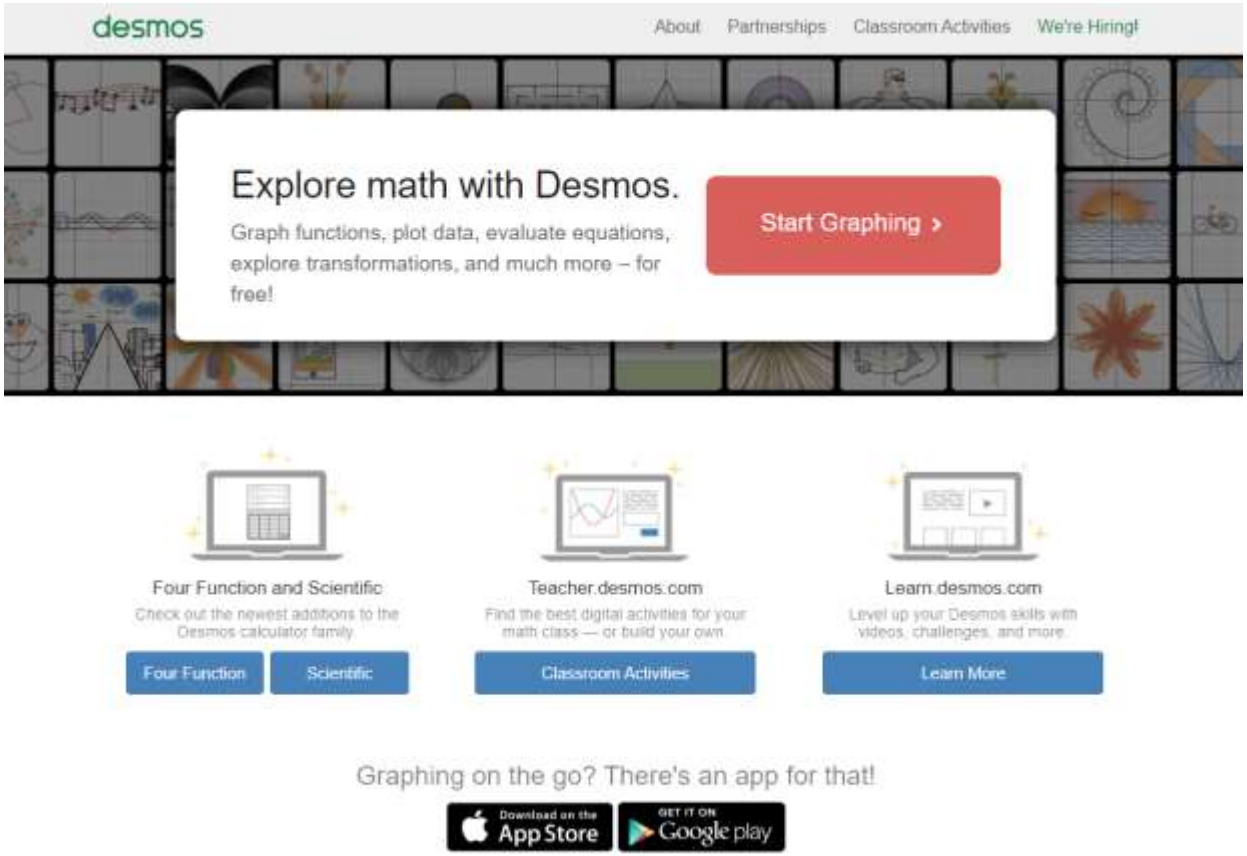
$g(x) = -x^2 + 6$

x	y
-3	3
0	6
4	-10



Desmos Graph

- https://www.desmos.com/

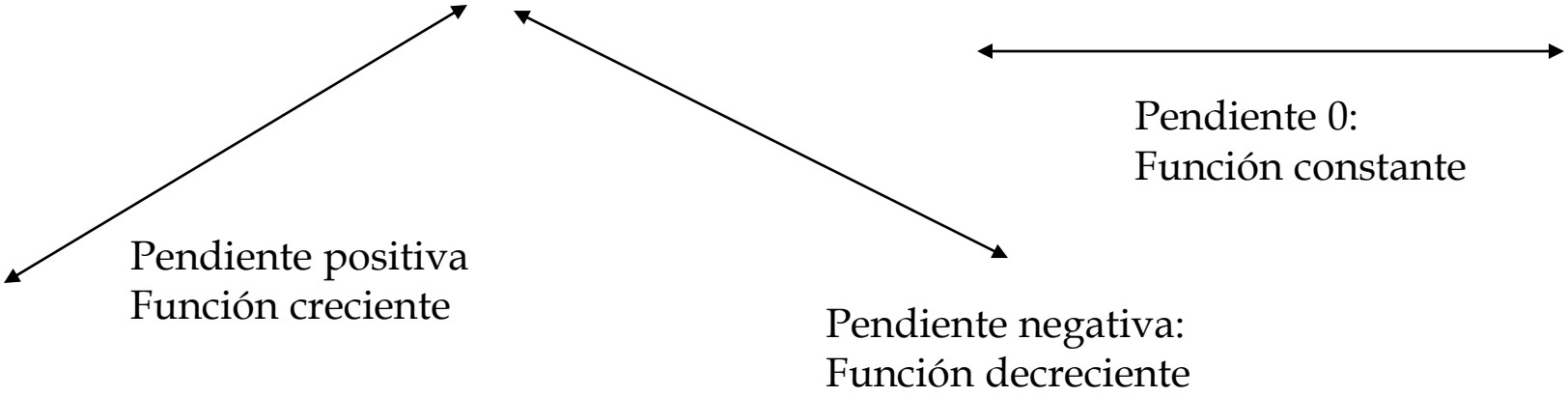


La Función Lineal

- La función lineal es la función de la forma:

$$f(x) = mx + b$$

- La gráfica de una función lineal es la recta con pendiente *m*, intercepto en y en (0,b).
- Tres tipos de funciones lineales:



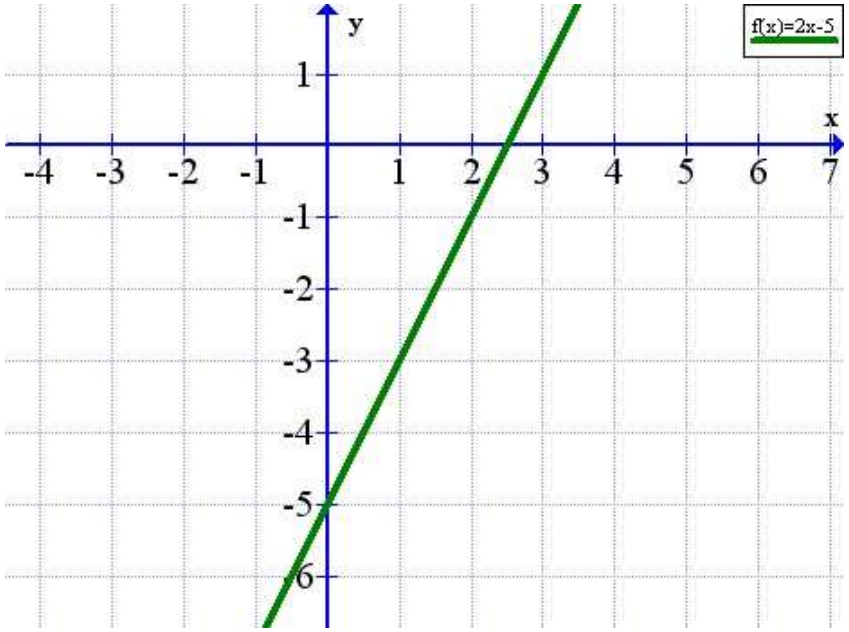


Ejemplo 2

- Ejemplos de gráficas de funciones lineales

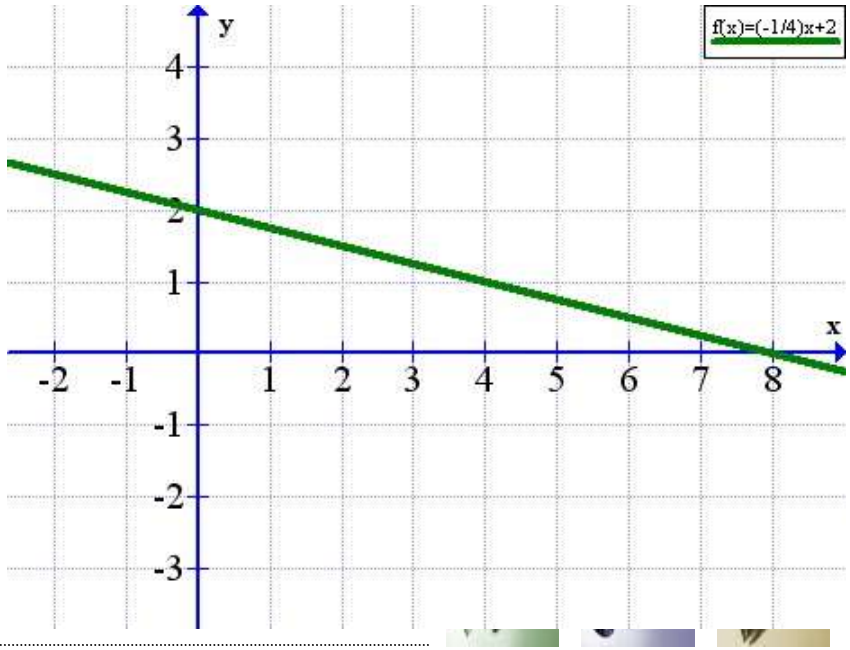
$$f(x) = 2x - 5$$

Pendiente = 2
Intercepto en y = (0, -5)



$$f(x) = -\frac{1}{4}x + 2$$

Pendiente = -1/4
Intercepto en y = (0, 2)

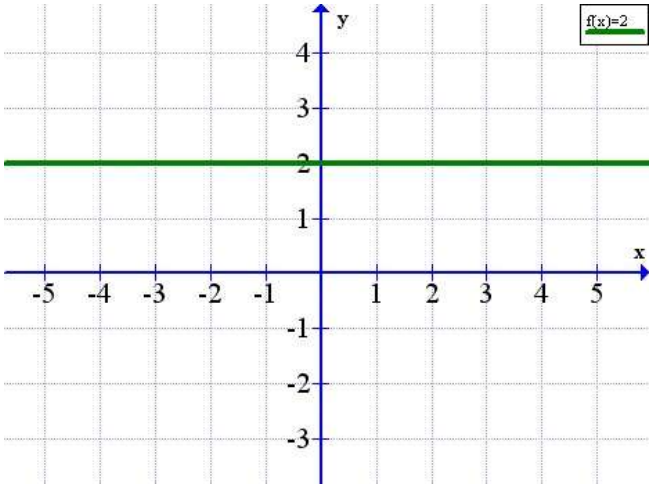


Función constante

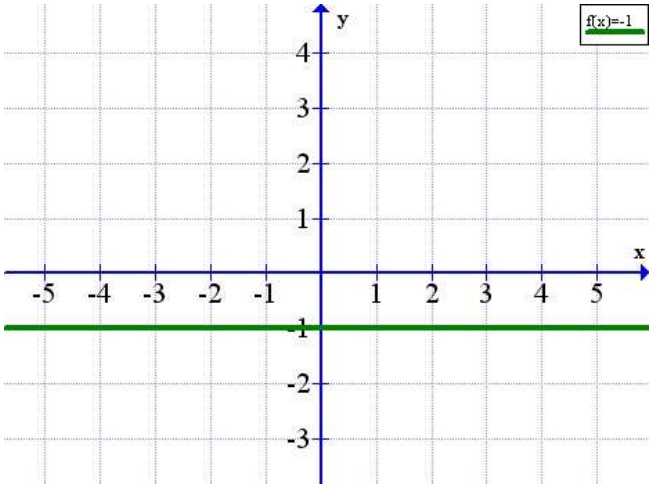
- Funciones lineales con pendiente 0

$$f(x) = número$$

$$f(x) = 2$$



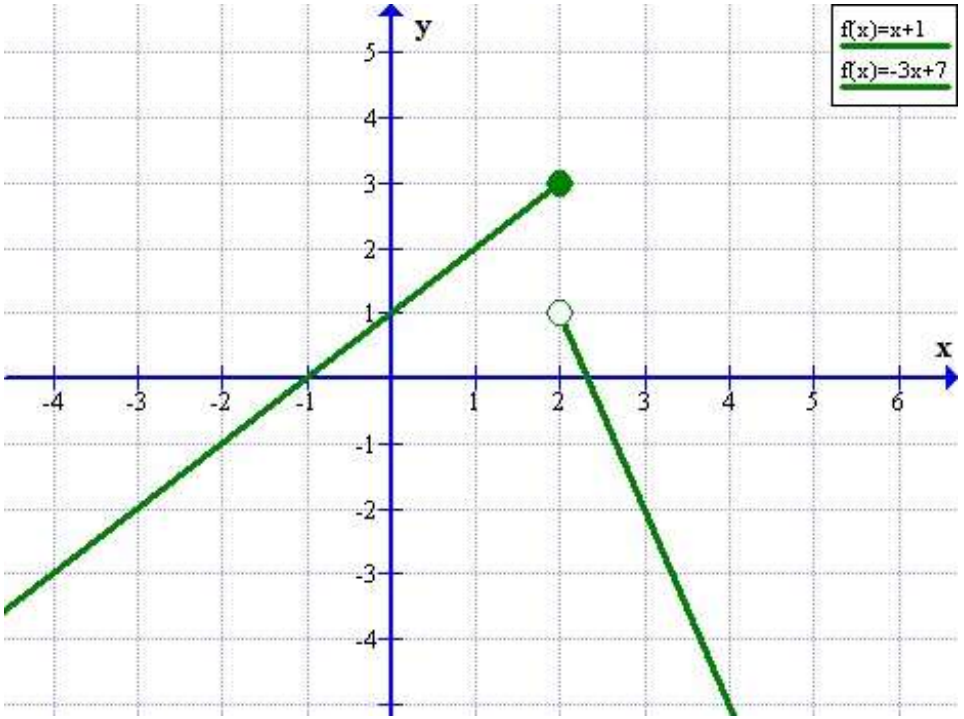
$$f(x) = -1$$





Funciones por tramo o con dominio dividido

- Son funciones cuyas gráficas consisten en segmentos.
- Si $f(x) = \begin{cases} x+1 & \text{si } x \le 2 \\ -3x+7 & \text{si } x > 2 \end{cases}$



- determine f(-2), f(2), f(3).

f(-2) = (-2) + 1 = -1

f(2) = -3(2) + 7 = 1

f(3) = -3(3) + 7 = -2

- Determine sus interceptos:

Si $y = f(x) = 0$

$0 = x + 1$ $\quad 0 = -3x + 7$

$x = -1$ $\quad 3x = 7$

$x = \frac{7}{3}$

Interceptos en x: $(-1,0), (\frac{7}{3}, 0)$

Si $x = 0$

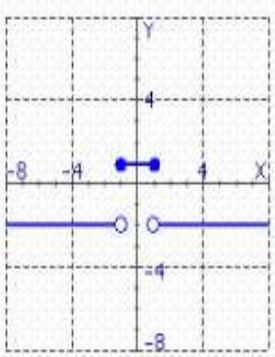
$f(0) = (0) + 1 = 1$

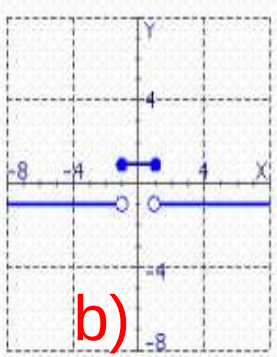
Interceptos en y: $(0, 1)$

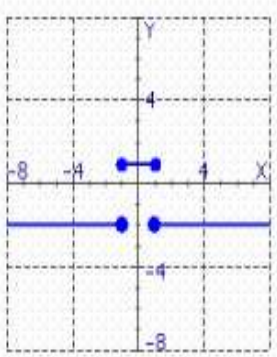


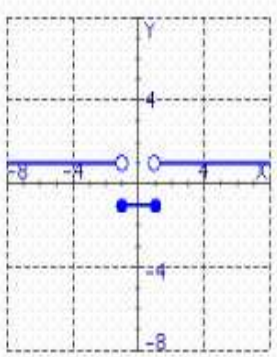
Ejemplo

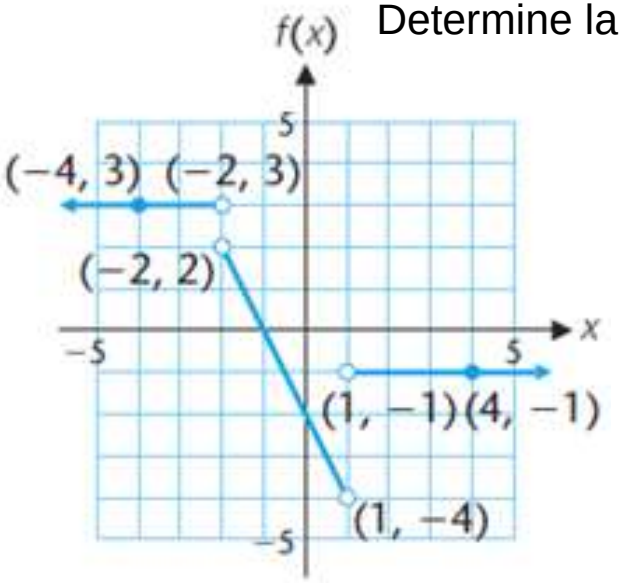
Seleccione la gráfica correcta de $f(x)=\begin{cases}-1 & \text{si} \quad x<-1\\ 1 & \text{si} \quad -1\le x\le 1\\ -1 & \text{si} \quad x>1\end{cases}$

a) 

b) 

c) 

d) 



Determine la función por partes cuya gráfica es:

Para $x<-2,\ y=3$

Para $-2<x<1,\ y=mx-2,\ \text{m}=\dfrac{-4-2}{1-(-2)}=\dfrac{-6}{3}=-2$

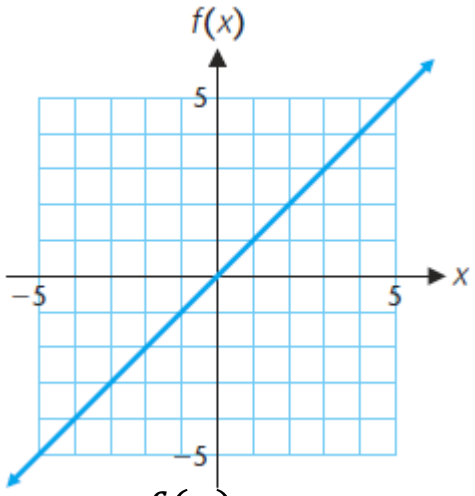
$y=-2x-2$

Para $x>1,\ y=-1$

$$f(x)=\begin{cases}3 & \text{si} \quad x<-2\\ -2x-2 & \text{si} \quad -2<x<1\\ -1 & \text{si} \quad x>1\end{cases}$$

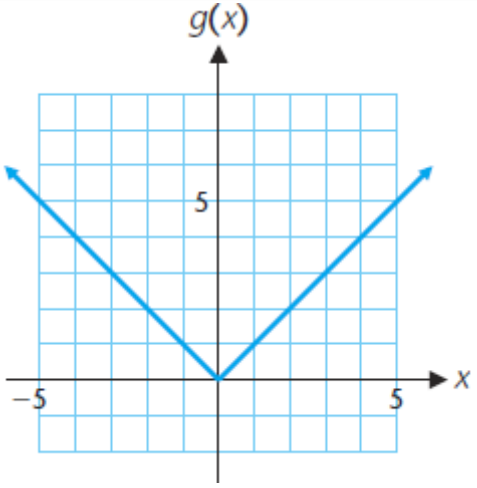


Funciones básicas



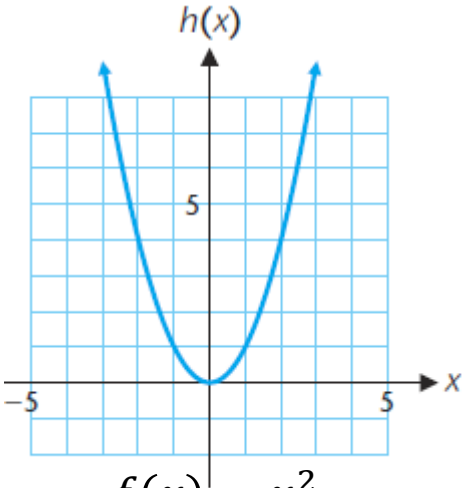
$f(x) = x$

Función identidad



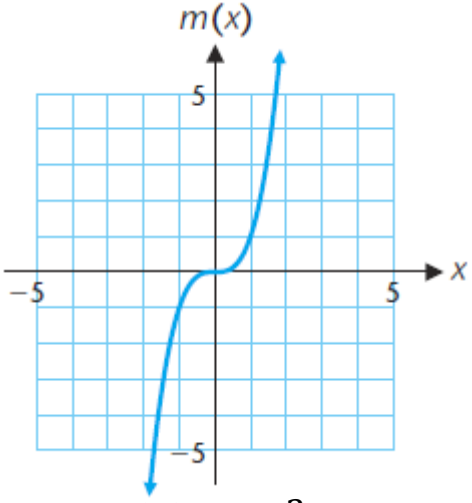
$f(x) = |x|$

Función Valor Absoluto



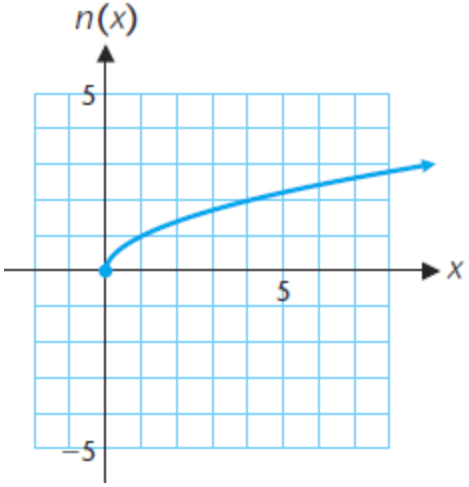
$f(x) = x^2$

Función Cuadrado



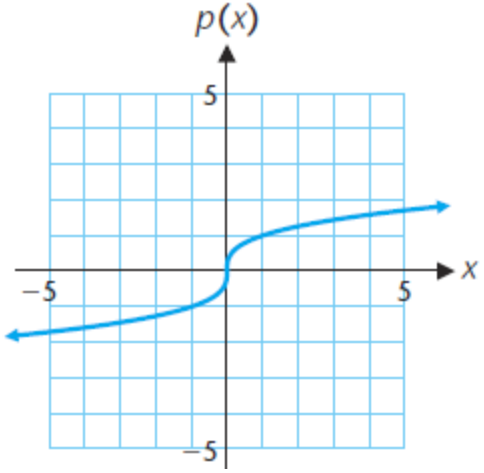
$f(x) = x^3$

Función Cúbica



$f(x) = \sqrt{x}|$

Función Raíz Cuadrada



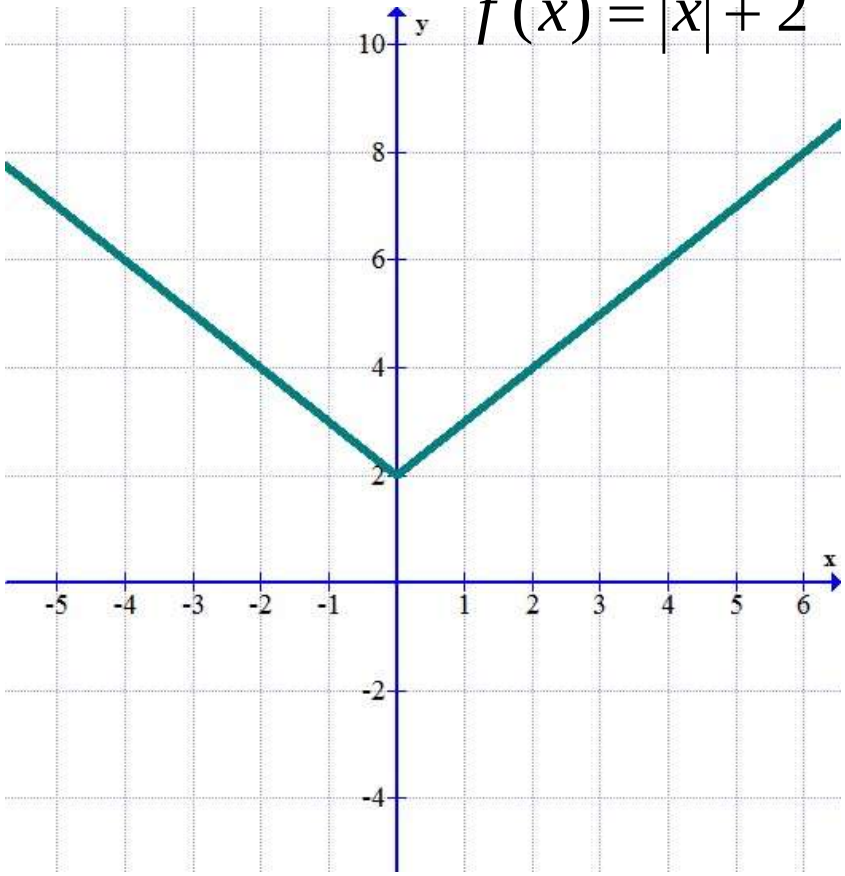
$f(x) = \sqrt[3]{x}|$

Función Raíz Cúbica

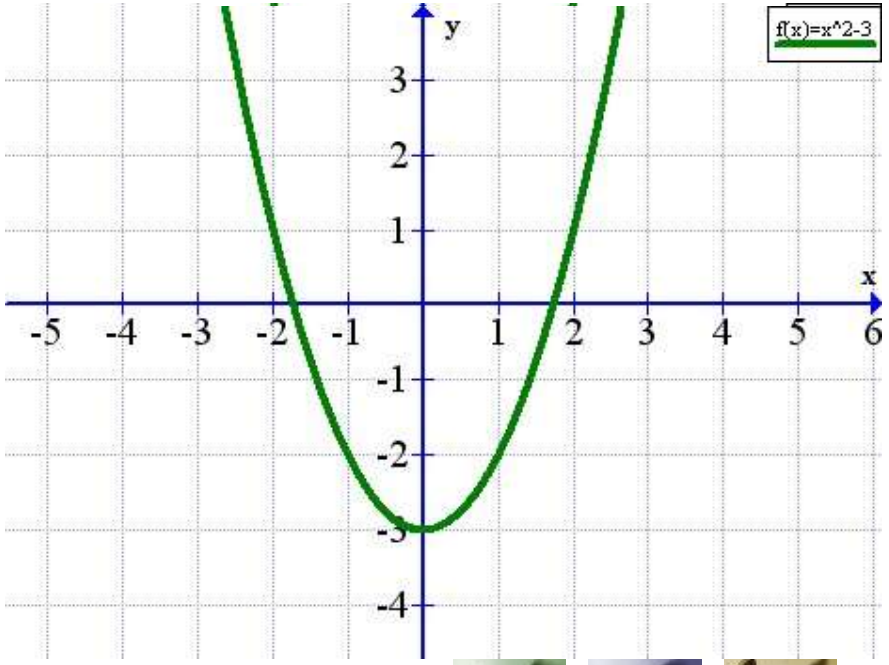
Desplazamiento vertical: $f(x)+a$

- Si a es un número reales distinto de 0, entonces, la gráfica de $f(x)+a$ es una traslación **vertical** de la gráfica de $f(x)$ por a unidades:

$f(x)=|x|+2$



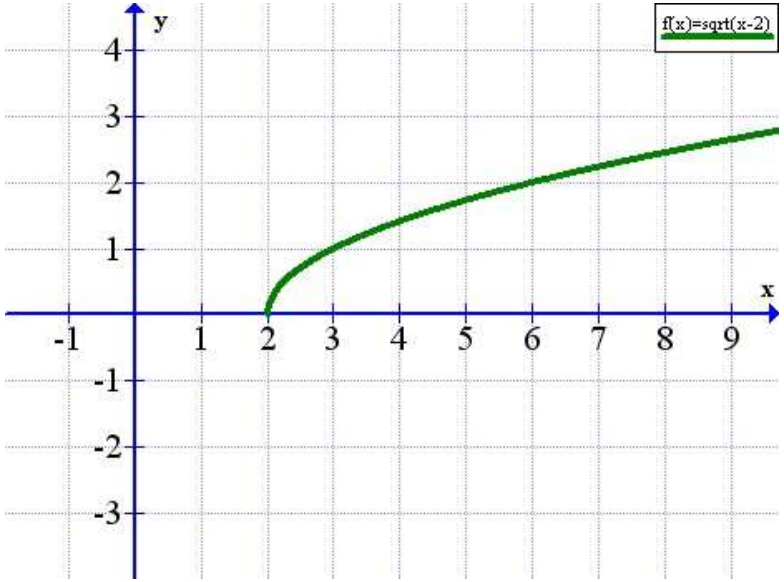
$f(x)=x^2-3$



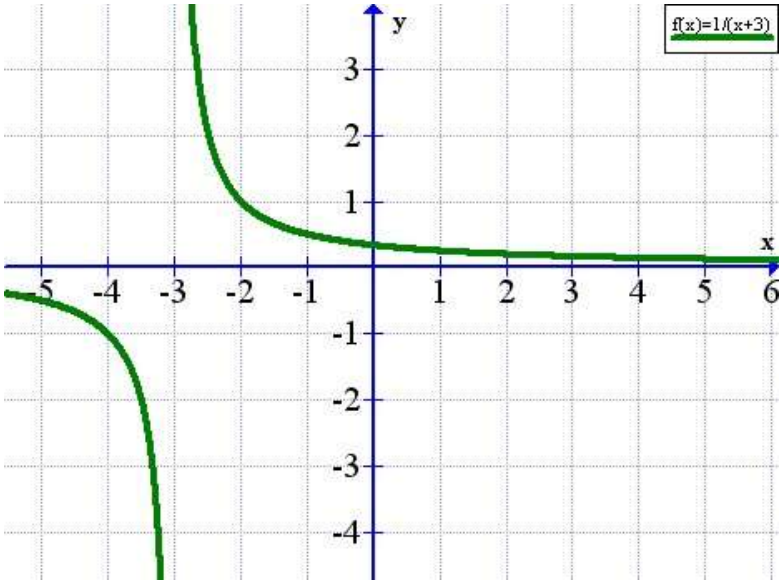
Desplazamiento horizontal: $f(x+a)$

Si a es un número reales distinto de 0, entonces la gráfica de $f(x+a)$ será una traslación **horizontal** de la gráfica de $f(x)$ por a unidades.

$$f(x)=\sqrt{x-2}$$



$$f(x)=\frac{1}{x+3}$$

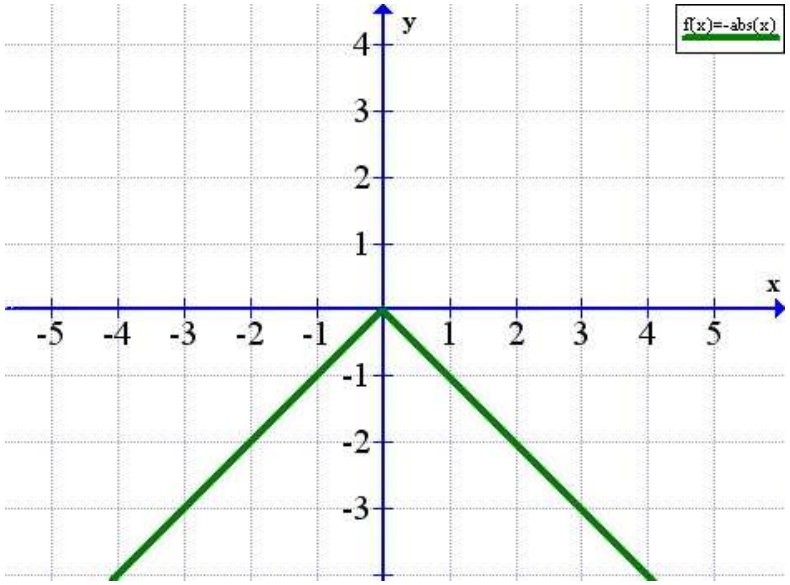




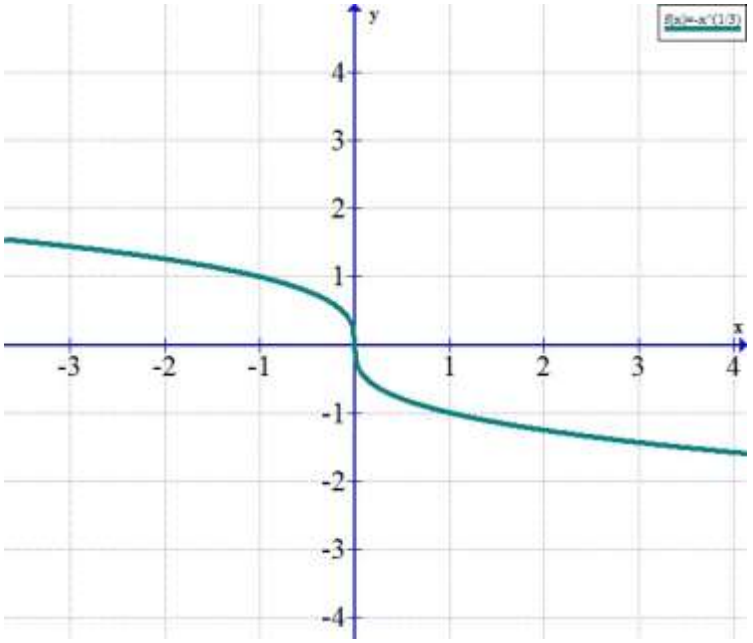
Reflexión horizontal: $-f(x)$

- La gráfica de - f(x) es una reflexión simétrica de f(x) con respecto al eje de x.

$$f(x) = -|x|$$



$$f(x) = -\sqrt[3]{x}$$



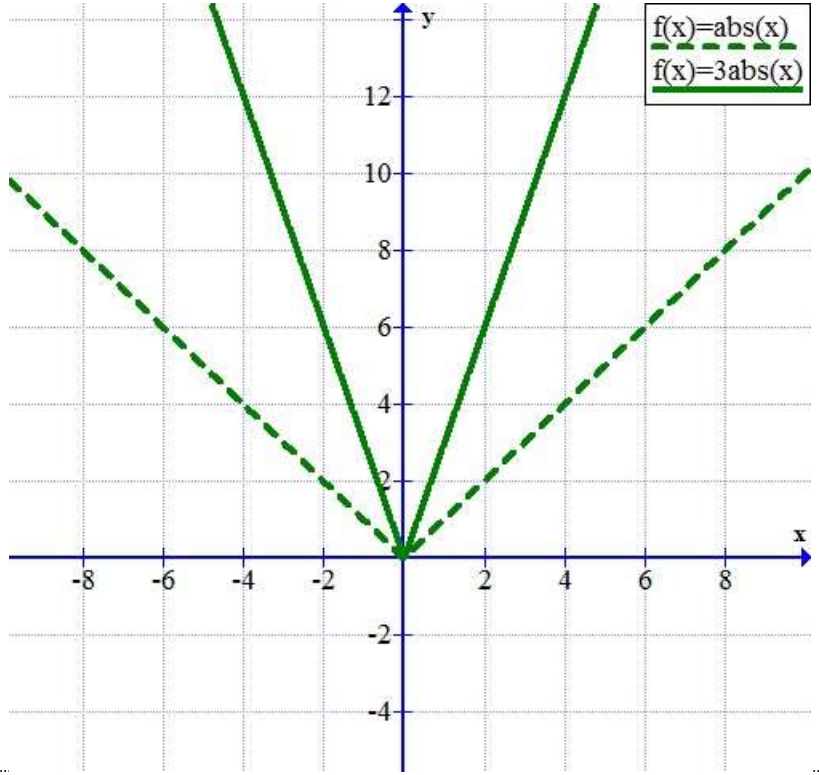


Estiramiento y compresión vertical: $af(x)$

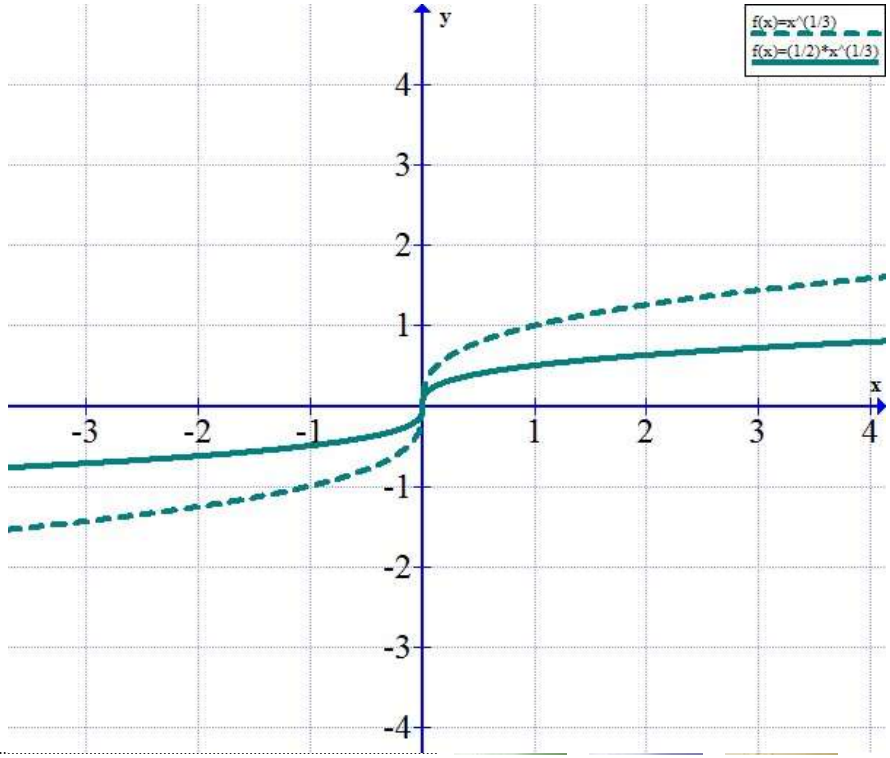
Sea a es un número real distinto de 0, entonces la gráfica de $af(x)$

- cuando $\boldsymbol{a > 1}$, será un estiramiento vertical de $f(x)$
- cuando $\boldsymbol{0 < a < 1}$, será una compresión vertical de $f(x)$

$$f(x) = 3|x|$$



$$f(x) = \frac{1}{2}\sqrt[3]{x}$$



Ejemplo 1

- Bosqueje la gráfica de la función $f(x) = \frac{x+2}{x-1}$

$$f(x) = \frac{x+2}{x-1}$$

$$= 1 + \frac{3}{x-1}$$

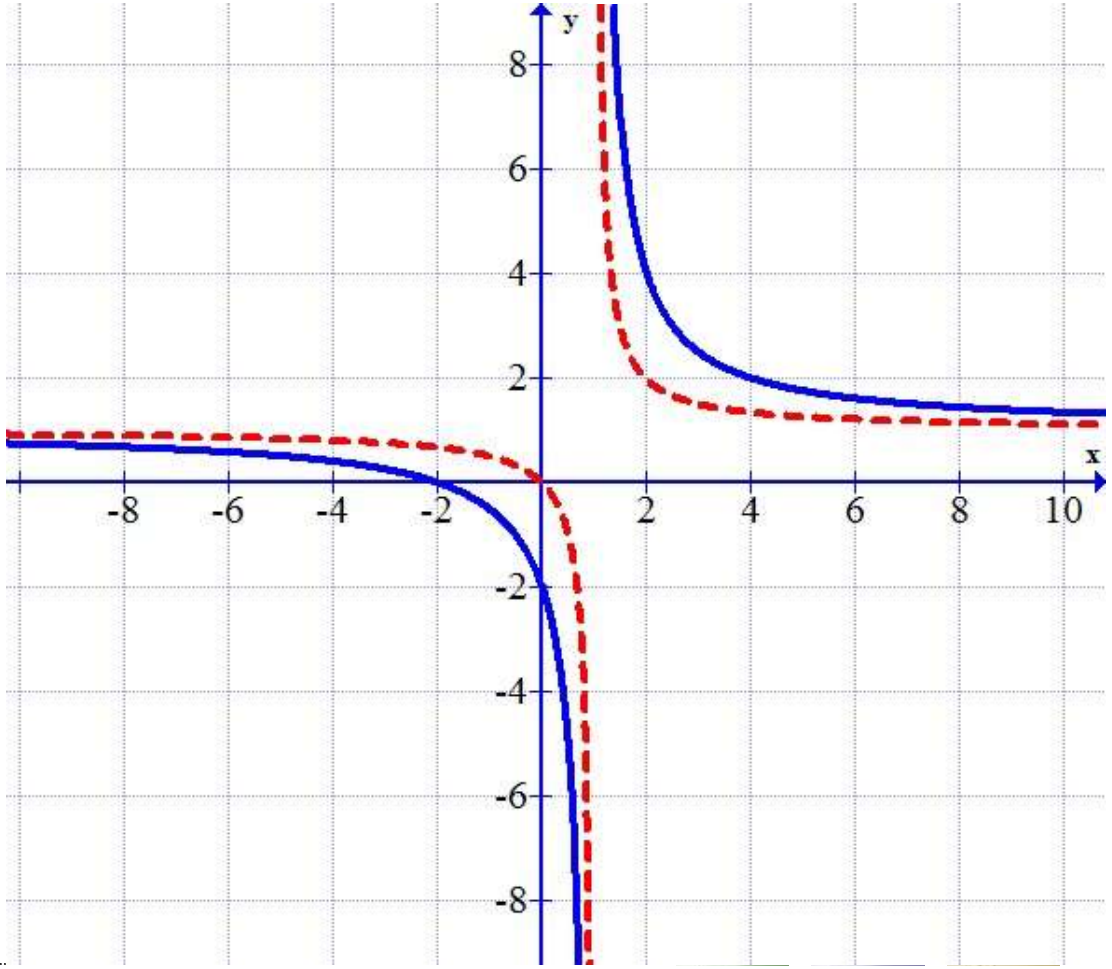
$$= \frac{3}{x-1} + 1$$

$$f(x) = \frac{1}{x}$$

$$f(x) = \frac{1}{x-1}$$

$$f(x) = \frac{1}{x-1} + 1$$

$$\boldsymbol{f(x) = \frac{3}{x-1} + 1}$$



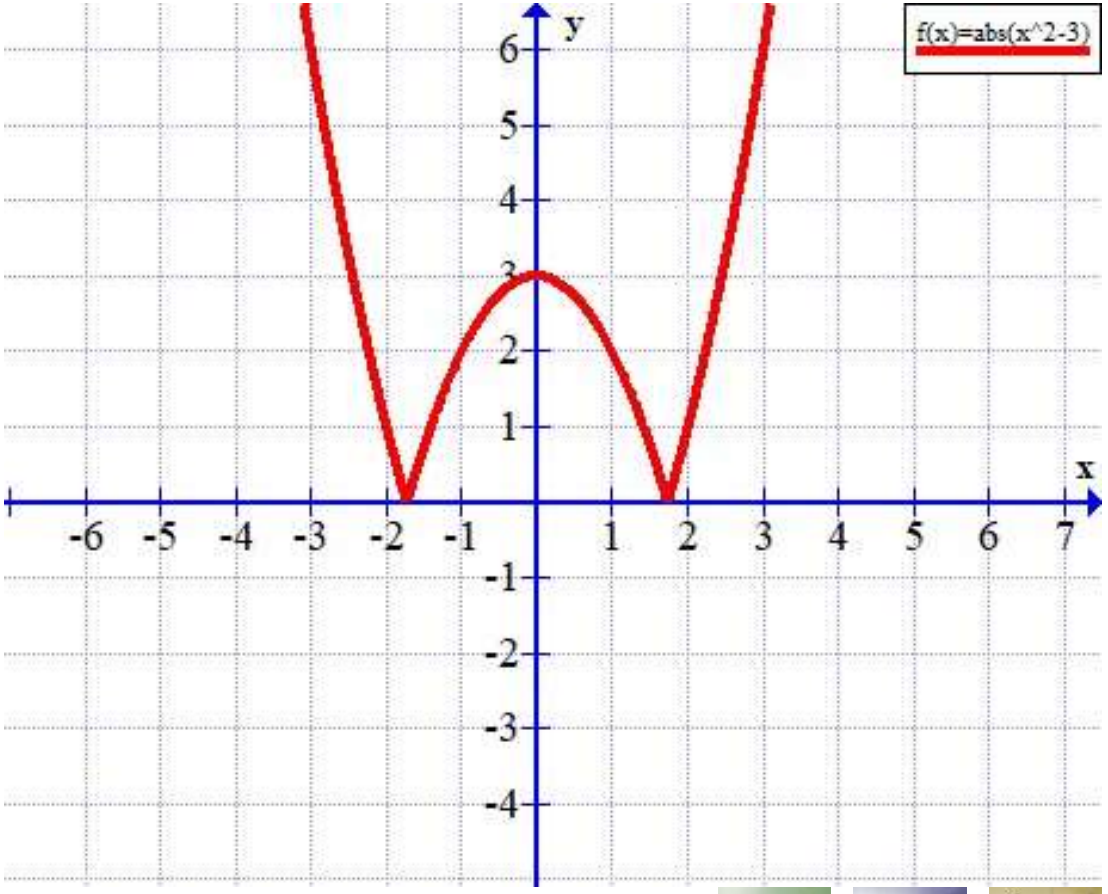
Ejemplo 2

- Bosqueje la gráfica de la función $f(x) = |x^2 - 3|$

$f(x) = x^2$

$f(x) = x^2 - 1$

$f(x) = |x^2 - 1|$



Funciones pares e impares

Sea f una función. Entonces,

- f es **par**, si para todo valor x

$$f(-x) = f(x)$$

  - Su gráfica es simétrica con respecto al eje de y

- f es **impar**, si si para todo valor x

$$f(-x) = -f(x)$$

  - Su gráfica es simétrica con respecto al punto origen



Ejemplo 3

- Determine si la función es par, impar o ninguna de las dos.

a) $f(x) = x^3 + x$

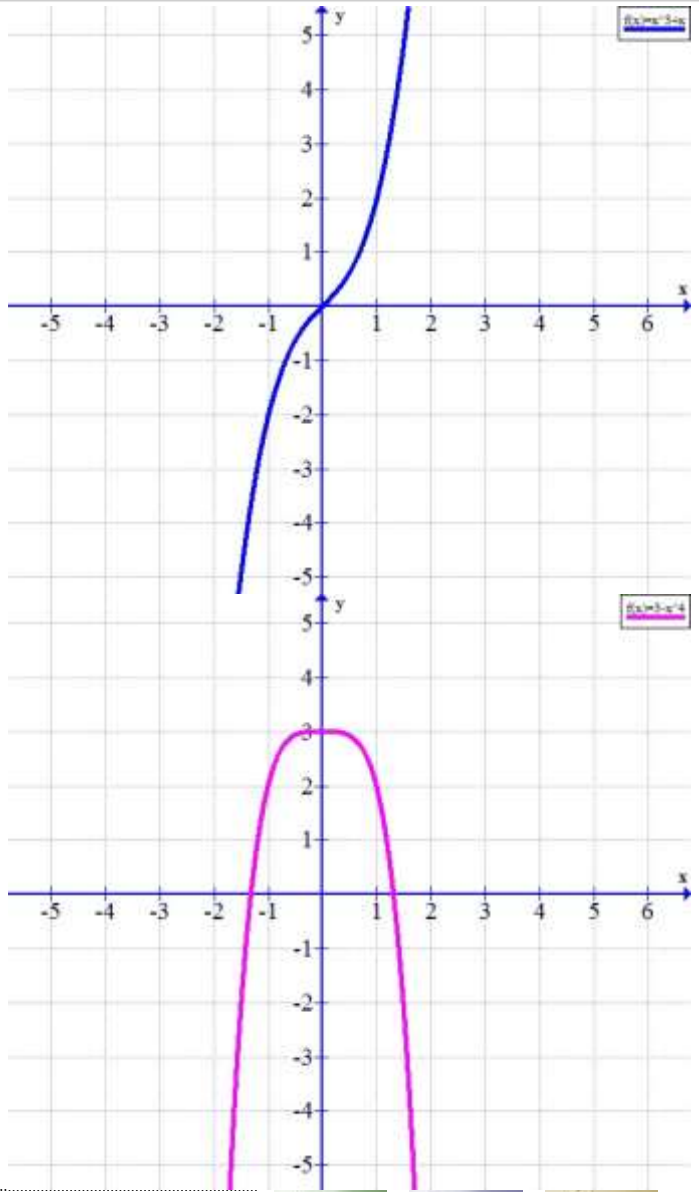
$$f(-x) = (-x)^3 + (-x)$$
$$= -x^3 - x$$
$$= -(x^3 + x) = -f(x)$$

Función es impar

b) $f(x) = 3 - x^4$

$$f(-x) = 3 - (-x)^4$$
$$= 3 - x^4$$
$$= f(x)$$

Función es par



Ejemplo 3 …

c) $f(x) = 5x - x^2$

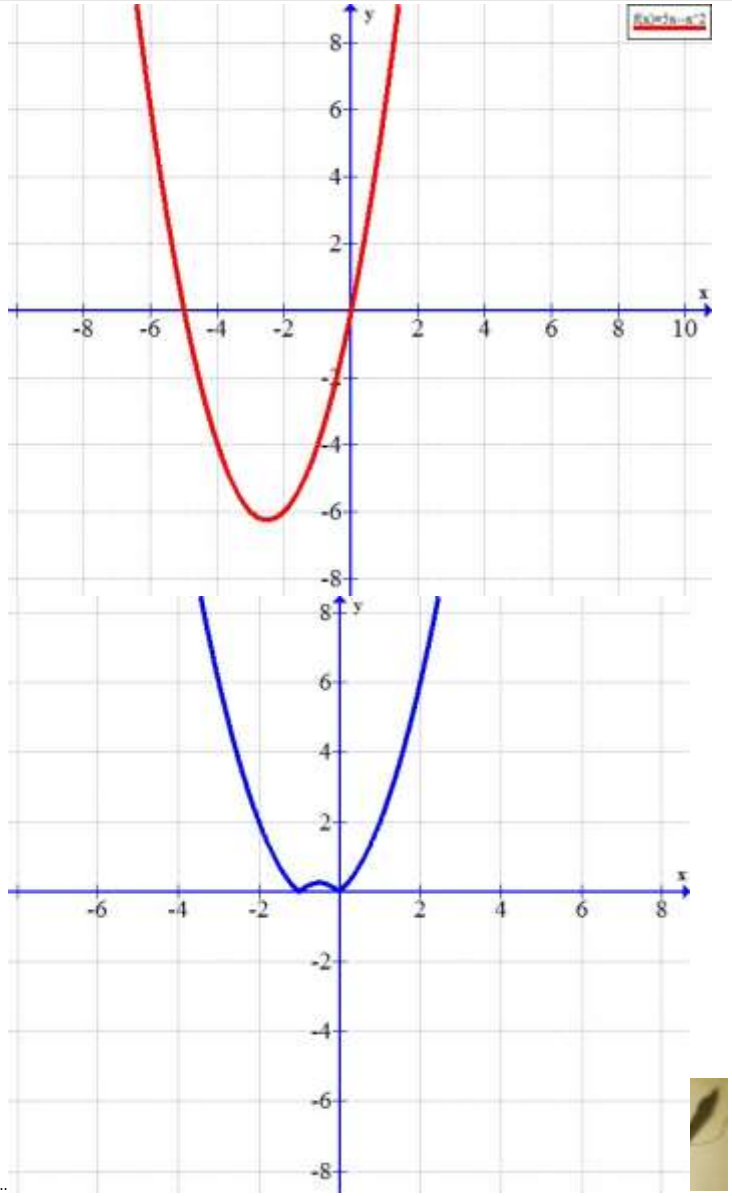
$$f(-x) = 5(-x) - (-x)^2$$
$$= -5x - x^2$$

Función no es ni par o impar

d) $f(x) = |x^2 + x|$

$$f(-x) = |(-x)^2 + (-x)|$$
$$= |x^2 - x|$$

Función no es ni par o impar



Ejercicios de Texto 2.2

2. If $f(4) = 10$ then the point $(4, \_\_\_\_)$ is on the graph of f.

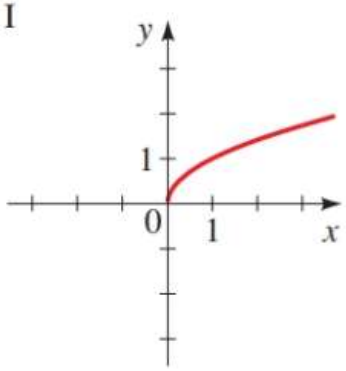
3. If the point $(3, 7)$ is on the graph of f, then $f(3) = \_\_\_\_\_$.

4. Match the function with its graph.

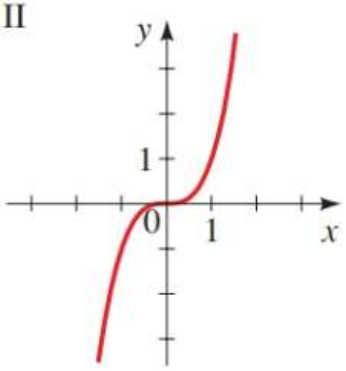
(a) $f(x) = x^2$ **(b)** $f(x) = x^3$

(c) $f(x) = \sqrt{x}$ **(d)** $f(x) = |x|$

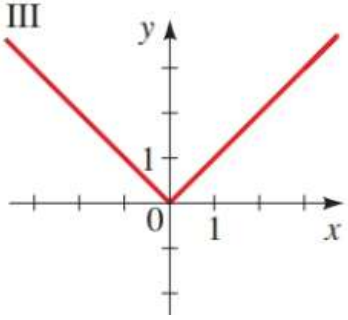
I



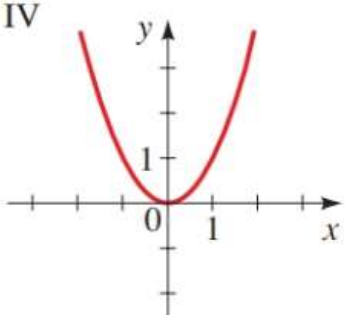
II



III



IV



5–28 ■ Graphing Functions Sketch a graph of the function by first making a table of values.

5. $f(x) = x + 2$

6. $f(x) = 4 - 2x$

7. $f(x) = -x + 3, \quad -3 \le x \le 3$

8. $f(x) = \dfrac{x - 3}{2}, \quad 0 \le x \le 5$

9. $f(x) = -x^2$

10. $f(x) = x^2 - 4$

33–46 ■ Graphing Piecewise Defined Functions Sketch a graph of the piecewise defined function.

33. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \ge 2 \end{cases}$

34. $f(x) = \begin{cases} 1 & \text{if } x \le 1 \\ x + 1 & \text{if } x > 1 \end{cases}$

35. $f(x) = \begin{cases} 3 & \text{if } x < 2 \\ x - 1 & \text{if } x \ge 2 \end{cases}$

36. $f(x) = \begin{cases} 1 - x & \text{if } x < -2 \\ 5 & \text{if } x \ge -2 \end{cases}$

37. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

38. $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ 3 - x & \text{if } x \ge -1 \end{cases}$

39. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 1 & \text{if } -1 \le x \le 1 \\ -1 & \text{if } x > 1 \end{cases}$

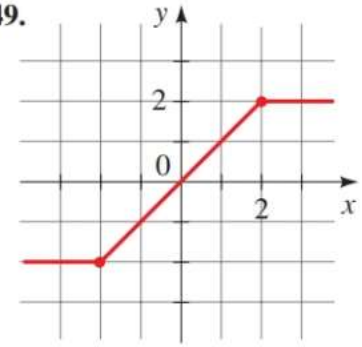
40. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$



Ejercicios de Texto 2.2

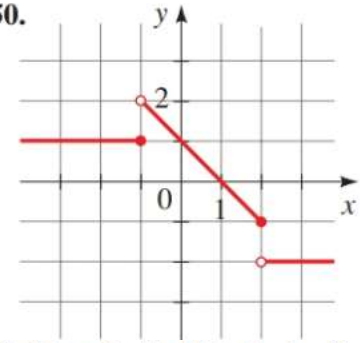
49–50 ■ Finding Piecewise Defined Functions A graph of a piecewise defined function is given. Find a formula for the function in the indicated form.

49.



$$f(x) = \begin{cases} \quad & \text{if } x < -2 \\ \quad & \text{if } -2 \le x \le 2 \\ \quad & \text{if } x > 2 \end{cases}$$

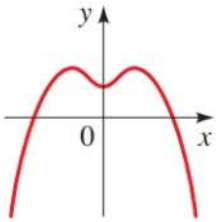
50.



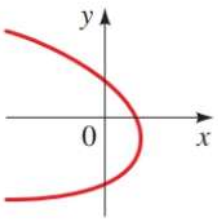
$$f(x) = \begin{cases} \quad & \text{if } x \le -1 \\ \quad & \text{if } -1 < x \le 2 \\ \quad & \text{if } x > 2 \end{cases}$$

51–52 ■ Vertical Line Test Use the Vertical Line Test to determine whether the curve is a graph of a function of x.

51. (a)

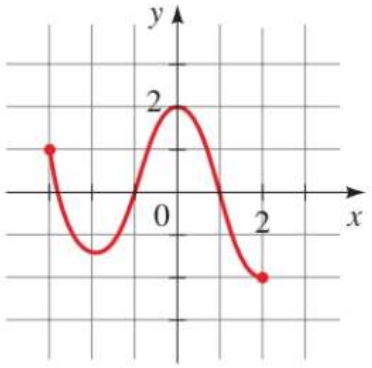


(b)



53–56 ■ Vertical Line Test: Domain and Range Use the Vertical Line Test to determine whether the curve is a graph of a function of x. If it is, state the domain and range of the function.

53.



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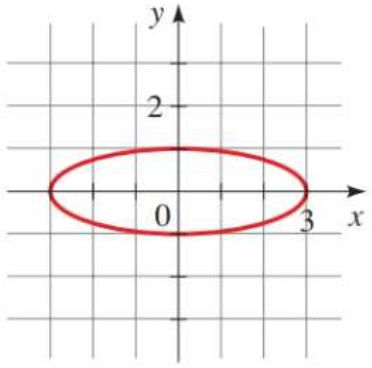

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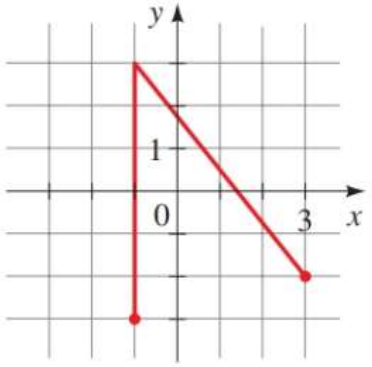

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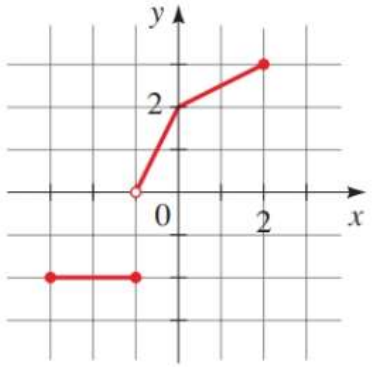


Ejercicios de Texto 2.2 & 2.3

57–68 ■ Equations That Define Functions Determine whether the equation defines y as a function of x. (See Example 9.)

57. $3x - 5y = 7$

58. $3x^2 - y = 5$

59. $x = y^2$

60. $x^2 + (y - 1)^2 = 4$

61. $2x - 4y^2 = 3$

62. $2x^2 - 4y^2 = 3$

63. $2xy - 5y^2 = 4$

64. $\sqrt{y} - x = 5$

65. $2|x| + y = 0$

66. $2x + |y| = 0$

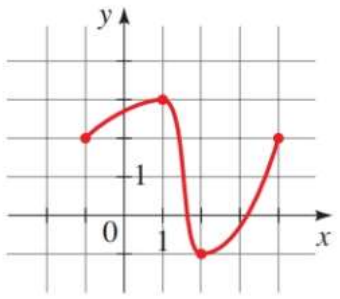
67. $x = y^3$

68. $x = y^4$

Sección 2.3

31–34 ■ Increasing and Decreasing The graph of a function f is given. Use the graph to estimate the following. **(a)** The domain and range of f. **(b)** The intervals on which f is increasing and on which f is decreasing.

31.



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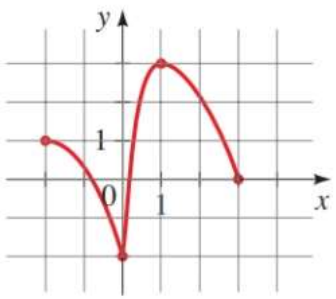
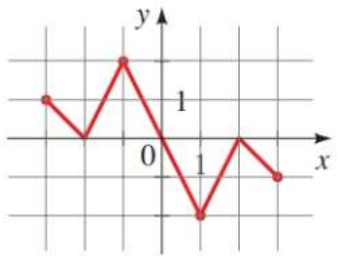
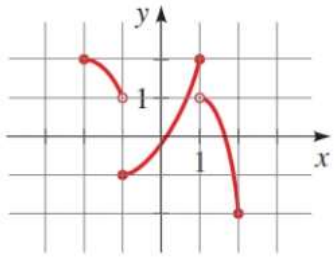


33.



34.



35–42 ■ Increasing and Decreasing A function f is given. **(a)** Use a graphing calculator to draw the graph of f. **(b)** Find the domain and range of f. **(c)** State approximately the intervals on which f is increasing and on which f is decreasing.

35. $f(x) = x^2 - 5x$

36. $f(x) = x^3 - 4x$

37. $f(x) = 2x^3 - 3x^2 - 12x$

Ejercicios de Texto 2.6

1–2 ■ Fill in the blank with the appropriate direction (left, right, up, or down).

1. **(a)** The graph of $y = f(x) + 3$ is obtained from the graph of $y = f(x)$ by shifting _____ 3 units.

(b) The graph of $y = f(x + 3)$ is obtained from the graph of $y = f(x)$ by shifting _____ 3 units.

2. **(a)** The graph of $y = f(x) - 3$ is obtained from the graph of $y = f(x)$ by shifting _____ 3 units.

(b) The graph of $y = f(x - 3)$ is obtained from the graph of $y = f(x)$ by shifting _____ 3 units.

3. Fill in the blank with the appropriate axis (x-axis or y-axis).

(a) The graph of $y = -f(x)$ is obtained from the graph of $y = f(x)$ by reflecting in the _____.

(b) The graph of $y = f(-x)$ is obtained from the graph of $y = f(x)$ by reflecting in the _____.

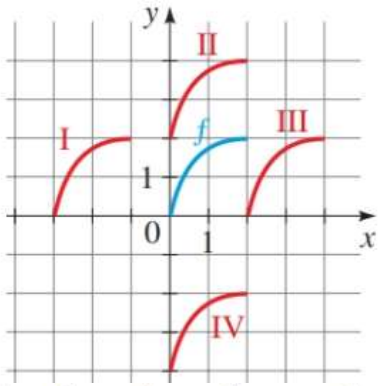
4. A graph of a function f is given. Match each equation with one of the graphs labeled I–IV.

(a) $f(x) + 2$

(c) $f(x - 2)$

(b) $f(x + 3)$

(d) $f(x) - 4$



7–18 ■ **Describing Transformations** Suppose the graph of f is given. Describe how the graph of each function can be obtained from the graph of f.

7. **(a)** $f(x) - 1$ **(b)** $f(x - 2)$

8. **(a)** $f(x + 5)$ **(b)** $f(x) + 4$

9. **(a)** $f(-x)$ **(b)** $3f(x)$

10. **(a)** $-f(x)$ **(b)** $\frac{1}{3}f(x)$

11. **(a)** $y = f(x - 5) + 2$ **(b)** $y = f(x + 1) - 1$

12. **(a)** $y = f(x + 3) + 2$ **(b)** $y = f(x - 7) - 3$

13. **(a)** $y = -f(x) + 5$ **(b)** $y = 3f(x) - 5$



Ejercicios de Texto 2.6 …

19–22 ■ Describing Transformations Explain how the graph of g is obtained from the graph of f.

19. **(a)** $f(x) = x^2, \quad g(x) = (x + 2)^2$
(b) $f(x) = x^2, \quad g(x) = x^2 + 2$

20. **(a)** $f(x) = x^3, \quad g(x) = (x - 4)^3$
(b) $f(x) = x^3, \quad g(x) = x^3 - 4$

21. **(a)** $f(x) = |x|, \quad g(x) = |x + 2| - 2$
(b) $f(x) = |x|, \quad g(x) = |x - 2| + 2$

22. **(a)** $f(x) = \sqrt{x}, \quad g(x) = -\sqrt{x} + 1$
(b) $f(x) = \sqrt{x}, \quad g(x) = \sqrt{-x} + 1$

23. Graphing Transformations Use the graph of $y = x^2$ in Figure 4 to graph the following.

(a) $g(x) = x^2 + 1$ **(b)** $g(x) = (x - 1)^2$
(c) $g(x) = -x^2$ **(d)** $g(x) = (x - 1)^2 + 3$

24. Graphing Transformations Use the graph of $y = \sqrt{x}$ in Figure 5 to graph the following.

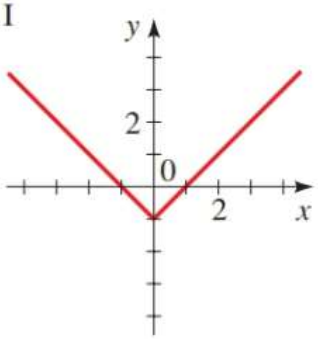
(a) $g(x) = \sqrt{x - 2}$ **(b)** $g(x) = \sqrt{x} + 1$
(c) $g(x) = \sqrt{x + 2} + 2$ **(d)** $g(x) = -\sqrt{x} + 1$

25–28 ■ Identifying Transformations Match the graph with the function. (See the graph of $y = |x|$ on page 96.)

25. $y = |x + 1|$ **26.** $y = |x - 1|$

27. $y = |x| - 1$ **28.** $y = -|x|$

I



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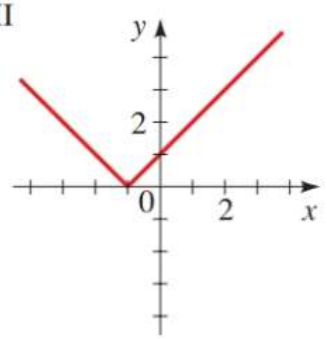
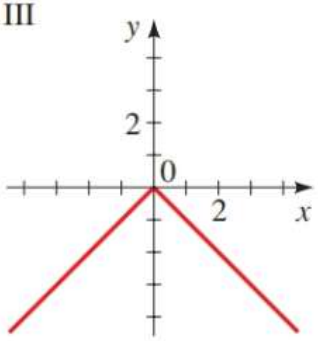
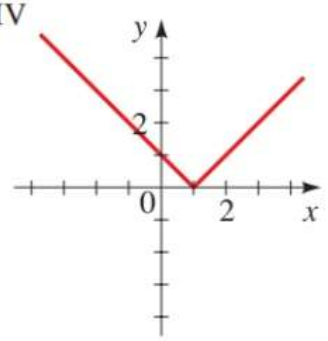


III



IV





Ejercicios de Texto 2.6 …

29–52 ■ Graphing Transformations Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

29. $f(x) = x^2 + 3$

30. $f(x) = x^2 - 4$

31. $f(x) = |x| - 1$

32. $f(x) = \sqrt{x} + 1$

33. $f(x) = (x - 5)^2$

34. $f(x) = (x + 1)^2$

35. $f(x) = |x + 2|$

36. $f(x) = \sqrt{x - 4}$

37. $f(x) = -x^3$

38. $f(x) = -|x|$

39. $y = \sqrt[4]{-x}$

40. $y = \sqrt[3]{-x}$

41. $y = \frac{1}{4}x^2$

42. $y = -5\sqrt{x}$

43. $y = 3|x|$

44. $y = \frac{1}{2}|x|$

45. $y = (x - 3)^2 + 5$

46. $y = \sqrt{x + 4} - 3$

47. $y = 3 - \frac{1}{2}(x - 1)^2$

48. $y = 2 - \sqrt{x + 1}$

49. $y = |x + 2| + 2$

50. $y = 2 - |x|$

51. $y = \frac{1}{2}\sqrt{x + 4} - 3$

52. $y = 3 - 2(x - 1)^2$

53–62 ■ Finding Equations for Transformations A function f is given, and the indicated transformations are applied to its graph (in the given order). Write an equation for the final transformed graph.

53. $f(x) = x^2$; shift downward 3 units

54. $f(x) = x^3$; shift upward 5 units

55. $f(x) = \sqrt{x}$; shift 2 units to the left

56. $f(x) = \sqrt[3]{x}$; shift 1 unit to the right

57. $f(x) = |x|$; shift 2 units to the left and shift downward 5 units

58. $f(x) = |x|$; reflect in the x-axis, shift 4 units to the right, and shift upward 3 units.

59. $f(x) = \sqrt[4]{x}$; reflect in the y-axis and shift upward 1 unit

60. $f(x) = x^2$; shift 2 units to the left and reflect in the x-axis

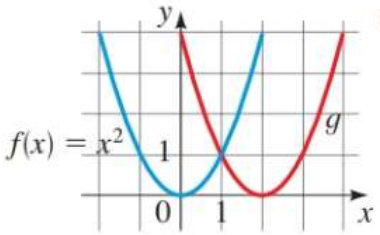
61. $f(x) = x^2$; stretch vertically by a factor of 2, shift downward 2 units, and shift 3 units to the right

62. $f(x) = |x|$; shrink vertically by a factor of $\frac{1}{2}$, shift to the left 1 unit, and shift upward 3 units

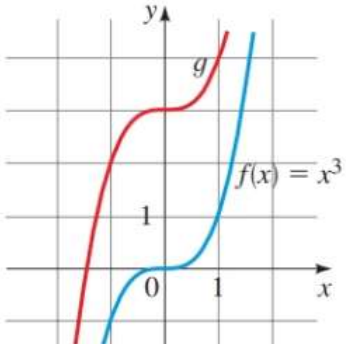
Ejercicios de Texto 2.6 …

63–68 ■ Finding Formulas for Transformations The graphs of f and g are given. Find a formula for the function g.

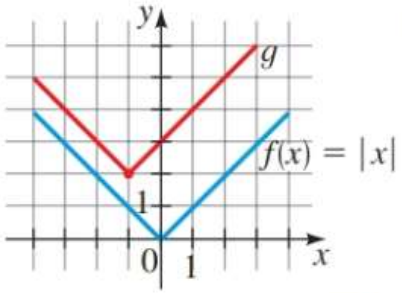
63.



64.



65.



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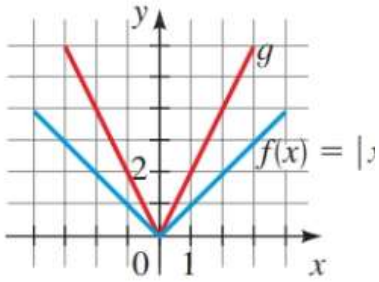
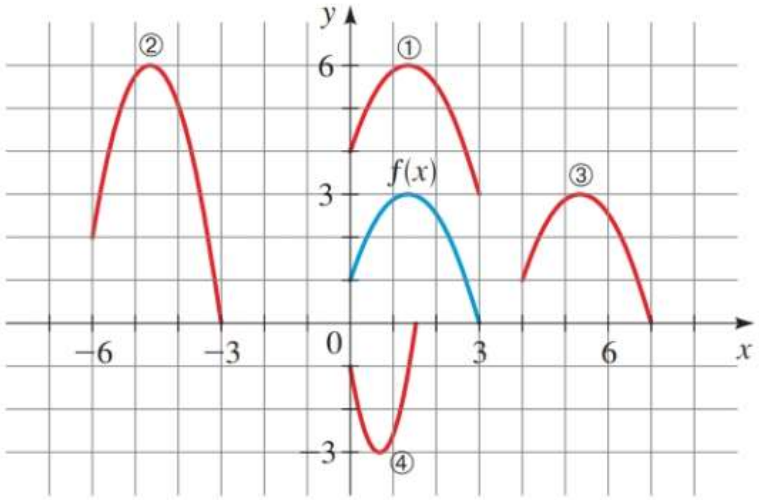


69–70 ■ Identifying Transformations The graph of $y = f(x)$ is given. Match each equation with its graph.

69. **(a)** $y = f(x - 4)$ **(b)** $y = f(x) + 3$
(c) $y = 2f(x + 6)$ **(d)** $y = -f(2x)$



83–90 ■ Even and Odd Functions Determine whether the function f is even, odd, or neither. If f is even or odd, use symmetry to sketch its graph.

83. $f(x) = x^4$

84. $f(x) = x^3$

85. $f(x) = x^2 + x$

86. $f(x) = x^4 - 4x^2$

87. $f(x) = x^3 - x$

88. $f(x) = 3x^3 + 2x^2 + 1$

89. $f(x) = 1 - \sqrt[3]{x}$

90. $f(x) = x + \dfrac{1}{x}$